CT System Parameter Calibration and Imaging Research

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Abstract: Computed tomography uses an X-ray or electron beam together with a highly sensitive detector to scan a section of the human body one after another, acquiring patient tomographic projection data and reconstructing the image using a computer. The topic takes the launch-receive model of parallel beam launch as the background, and discusses the basic working parameters of CT instrument, CT image reconstruction, and the absorption rate of the medium which is measured. Therefore, we use the method of analytic geometry and algebra to obtain the rotation center of the CT machine, the interval and the rotation angle of unit receiving (transmitting) device, fully exploiting the given data, and further utilizing the Radon transform and its inverse transform, the dimensionality reduction of the matrix and the method of fitting, mirroring of data to use the existing data to restore the original model and adjust the direction so that it conforms to the known relative position given in the question and obtains the basic information of the two unknown figures. Finally, on the premise of improving accuracy and stability, we optimize this model and propose a model that is closer to the actual operation that is, using Sector beam to make it more realistic.

1. Introduction

Computed tomography (CT) [1] can scan a part of an object with X-rays and highly sensitive detectors, and use the object's absorption characteristics of ray energy to image the object and obtain objects‘internal structural information. We tend to analyze a typical two-dimensional CT system in which X-rays are incident in parallel and perpendicular to the detector plane, and each detector unit is seen as a receiving point and arranged equidistantly.

The relative position of the X-ray emitter and detector is fixed, and the entire transmitting-receiving system is rotated counterclockwise 180 times around a fixed center of rotation. For each X-ray direction, the radiant energy absorbed by the two-dimensional to-be-detected medium with fixed position is measured on a detector with 512 equidistant elements, and 180 sets of received information are obtained after processing by gain and the like. When the CT system is installed, there are often errors, which affect the imaging quality. Therefore, it is necessary to calibrate the installed CT system, that is, to calibrate the parameters of the CT system by means of a sample of known structure which is called a template, and image the sample of the unknown structure [2].

The topic takes the launch-receive model of parallel beam launch as the background, and discusses the basic working parameters of CT instrument, CT image reconstruction, and the absorption rate of the medium which is measured. Therefore, we use the method of analytic geometry and algebra to obtain the rotation center of the CT machine, the interval and the rotation angle of unit receiving (transmitting) device, fully exploiting the given data, and further utilizing the Radon transform [3] and its inverse transform, the dimensionality reduction of the matrix and the method of fitting, mirroring of data to use the existing data to restore the original model and adjust the direction so that it conforms to the known relative position given in the question and obtains the basic information of the two unknown figures [4]. Finally, on the premise of improving accuracy and stability, we optimize this model and propose a model that is closer to the actual operation that is, using Sector beam to make it more realistic.

The problem solved in this paper comes from the title of the undergraduate group of the 2017 National College Student Mathematical Modeling Contest in 2017. The questions “Question 1”, “Question 2” and “Attachment 1” and “Attachment 2” mentioned in the article, please read the information of the question

2. CT system parameter calibration model

2.1 Distance between detector units

In 1990, Gullberg proposed a calibration method for cone-beam CT systems, which collects equally-spaced projection data of point targets over a range of angles, and then calculates [5]. In 1994, Rizo proposed a corresponding improvement method, which divides the geometric parameters of the solution into internal parameters and external parameters. The internal parameters refer to the system parameters that can be directly measured, including the distance from the source to the detector, the Coordinates of center ray, external parameters refer to parameters related to rotational motion. According to detection environment and background of the CT given by the title, the detector is divided into 512 units, then we regard the X-ray in each unit as a light. Continuing to analyze the matrix data distribution of 512*180 in Annex 2, we found that when the template is illuminated from 180 directions with parallel X-rays, the longer the template passes through each ray, the larger the value of the accepted information in the corresponding matrix [7-8].

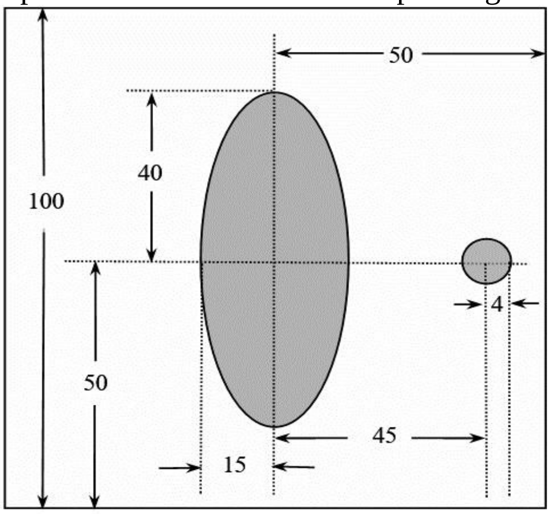

Figure 1. Information of the template geometry

We analyzed the geometric information of the template given by the title. The template consists of an ellipse and a circle. The properties of the ellipse and the circle show that when the ray is parallel to the long axis of the ellipse, the light passing through the long axis of the ellipse corresponds is the maximum value of the data in the matrix. Traverse through Annex 1, find its maximum value and position, get its maximum value is 141.7749, position coordinates are [x1, y1] = [233, 151].

Use the matlab program to find the maximum value and its position in the matrix, we can get the maximum value of 141.7749. The position of the maximum value in the matrix [x1, y1] = [233, 151], then the 512 values in the 151st column are obtained when the ray is parallel to the long axis of the ellipse. It is not difficult to find that there are two maxima in the 151st column. The rays corresponding to the two maxima are the light passing through the center of the circle and the light passing through the center of the ellipse. According to the nature of the maxima, it can be found by program. The position of the corresponding value of the light at the center of the ellipse is given above, [x1, y1] = [233, 151], and the position of the corresponding value of the other ray is [x2, Y2]=[60,151]. That is to say, the two rays differ by $\Delta x = x_1 - x_2 = 163$ detector units, which can be obtained from Fig. 1, and the distance between the center of the ellipse and the center of the circle is d0=45 mm. So the distance between the detector units.

$$d = d_0 / \Delta x = 0.2761 mm \tag{1}$$

2.2 180 directions of X-rays

In the above we have obtained the number of data columns corresponding to the direction parallel to the long axis of the ellipse. We continue to analyze the geometric properties of the template. We find that when the X-ray is parallel to the short axis of the ellipse, the number of rays that can pass through the template is the largest. So, we wrote a program to find out the columns with the most non-zero data. The receiving plate of the detector is numbered 1-512, corresponding to the abscissa of the image in figure. The abscissa of the smaller value of the smaller value in figure 2 is smaller than the abscissa of the maximum value of the larger value, so when the detector rotates to the direction, the receiving plate is above the figure 2 and the label is incremented from right to left.

When the detector rotates to an angle, as shown in the figure, if the angle between the X-ray and the y-axis is θ, the distance between the two tangent lines in the figure is 1, and the corresponding column vector is removed from the head and tail. The number of zeros is n, so

$$l = n * d \tag{2}$$

It is possible to calculate n(i), (i = 1, 2, 3, ... 180) of 180 sets of data. In this way, 180 sets of data about distance 1 can be calculated and set to l(i), (i = 1, 2, 3, ... 180). On the other hand, calculating l from the aspect of analytic geometry. According to our guess, the initial rotation angle of the detector is θ=90-61=29. Since there are 180 directions and the step size is 1 degree, θ = 29:1:219 is calculated. Taking the situation in the figure as an example, the equation of the ellipse is

$$\frac{x^2}{225} + \frac{y^2}{1600} = 1 \tag{3}$$

The equation of the line tangent to the ellipse is:

$$y = -\frac{1}{\tan\theta} x + b_1 \tag{4}$$

The value of b1 can be immediately solved combing the two equations.

The equation for the small circle is:

$$(x-45)^2 + y^2 = 16 \tag{5}$$

The tangent equation is:

$$y = -\frac{1}{\tan\theta} x + b_2 \tag{6}$$

The values of b2 are obtained immediately by combining the two equations. At this point, the distance between the corresponding two tangent lines is:

$$l'(i) = (b_2 - b_1) * \sin\theta \tag{7}$$

In other cases, a similar method can be used to obtain $l'(i), i = 1,2,\ldots 180$. At this time, using the corr2 function in MATLAB to compare the similarity with the two matrices, the similarity is 0.999986, so We think that the 180 directions of the X-rays used by the CT system correspond to $\theta = 29:1:219$.

2.3 Center of rotation of the CT system

Next we use the 61st and 151st columns to calculate the center of rotation of the CT system. At this time, we re-establish the Cartesian coordinate system with the lower left corner of the square tray in Figure 1 as the origin. When the detector rotates to the direction corresponding to the data in column 151, the labels on the absorption plates corresponding to the center of the ellipse and the center of the small circle are 223 and 60 respectively. According to the geometric data of the template, assuming that the coordinates of the two points on the absorption plate be (50, y1), (95, y1). Next, obtain the ordinates of the points on the plates labeled 223 and 60 when the detector is rotated to the direction corresponding to the data in the 61st column. Because the label on the absorption plate corresponding to the center of the small circle is 235, $y\_\{223\} = 46.6871$ $y\_\{60\} = 1.6871$. Let the coordinates of the two points on the absorption plate be $(x_{1,}46.6871), (x_{1,}1.6871)$.

The following equation can be obtained by equating the distances of the receiving plates from the center of rotation to each direction and the distances from the center of rotation to the same number in all directions:

$$\begin{gathered} x_1 - x_0 = y_1 - y_0 \\ \sqrt{(x_0 - x_1)^2 + (1.6871 - y_0)^2} = \sqrt{(x_0 - 95)^2 + (y_0 - y_1)^2} \\ \sqrt{(x_0 - 50)^2 + (y_1 - y_0)^2} = \sqrt{(x_0 - x_1)^2 + (y_0 - 46.6871)^2} \end{gathered} \tag{8}$$

The following proves that the label on the receiving plate corresponding to the center of rotation is 256 regardless of the direction in which the detector is rotated. In the above we have shown that the angle between the first rotation direction and the 180th rotation direction is 180, that is, the receiving plates are parallel to each other in these two directions. Pick a special point: the center of the small circle.

In the first direction, the label on the receiving board corresponding to the center of the small circle is 416, and the label on the receiving board corresponding to the center of the small circle in the 180th direction is 95, since 416+95+1=512, equaling to the total number of labels on the receiving board, so no matter what direction the detector rotates, the label on the receiving board corresponding to the center of rotation is 256 (the center position of the receiving board). Therefore, the abscissa of the center of rotation can calculate the abscissa of the center of rotation by using the difference between the index of the ellipse center and the center of rotation in the direction corresponding to the 151th data. In summary, in the Cartesian coordinate system set in this section, the coordinates of the center of rotation are (40.8896,55.7975).

3. CT image reconstruction model

3.1 Problem background

The key to Problem 2 is to reconstruct the position, geometry, and absorption rate of the unknown template based on the existing attachments 2, 3 and the location of the known template. So the breakthrough is in the relationship between the geometry of the template and the energy absorption. So we quoted the Radon transform here. Radon essentially is an integral transformation

$$T(\rho,\theta) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\delta(\rho - x\cos\theta - y\sin\theta) \tag{9}$$

The Radon transform is a mathematical transformation method that transforms the original signal into the Radon domain along a path integral. This method has the advantage of good anti-noise performance and is widely used in medical imaging. The Radon transform is to make a projection transformation of the digital image matrix in the direction of a specified angle of ray, which means that we can do Radon transformation along any angle θ, such as

$$\rho = x\cos\theta + y\sin\theta \tag{10}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{11}$$

But in this problem, what we need is the inverse radon transform, because we know that the energy data is absorbed, and the unknown model needs to be solved. The iRadon function is a filtered back projection method based on an R-L filter.

3.2 Image reconstruction

CT reconstruction is generally divided into two major categories: analytical method and iterative method. Convolution back projection algorithm (CBP) in analytical method has a wide range of applications. It is the key to choice of the filter function in CT reconstruction, and the role of the filter is to eliminate artifacts in the back projection transformation. The advantage of the R-L filter is that the form is simple and the reconstructed image is clearer [9]. The Fourier function of the R-L filter function is multiplied by the projection of each angle in the frequency domain to achieve filtering [10]. If other is selected, the Fourier function of the R-L filter function is multiplied by the set function and multiplied by the projection of each angle in the frequency domain.

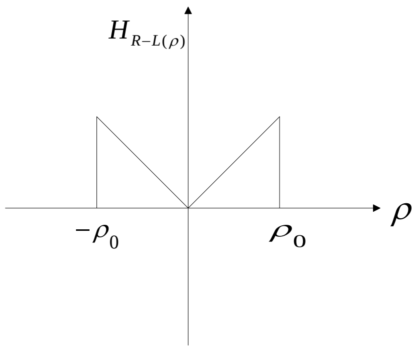

Figure 2. R-L filter frequency response diagram

The frequency domain function of the R-L filter is:

$$H_{R-L}(\rho) = |\rho| W(\rho) = |\rho| rect(\frac{\rho}{2B}) \tag{12}$$

In the formula:

$$rect(\frac{\rho}{2B}) = \begin{cases} 1, |\rho| < B = \frac{1}{2d} \\ 0 \end{cases} \tag{13}$$

Among them, ρ is the spatial frequency, $W(\rho)$ is a window function.

Corresponding time domain convolution function:

$$h_{R-L}(x_r) = 2B^2 \sin c(2x_r B) - B^2 \sin c^2(x_r B) \tag{14}$$

$x_r = nd$ will be brought into the form, the resulting of $h_{R-L}(x_r)$ discrete form:

$$h_{R-L}(x_r) = \begin{cases} \frac{1}{4d^2} & n = 0 \\ 0 & n = even \\ -\frac{1}{n^2\pi^2 d^2} & n = \text{odd} \end{cases} \tag{15}$$

According to the energy absorption data in Schedule 2, the original image can be reconstructed. Because the shape and position of the template are known, the position and geometry of the unknown medium can be restored by comparing the restored image with the original image and using the energy absorption data of the attached table. The following image shows the original image reconstructed using Annex 2:

After the iRadon transformation, it was found that the reconstructed model was tilted to the right compared to the original model. From the above, the initial illumination angle is 29 degrees from the long axis of the ellipse, so we assume that the tilt of the model is related to the initial angle. In order to align the model, we used the mirroring of the data.

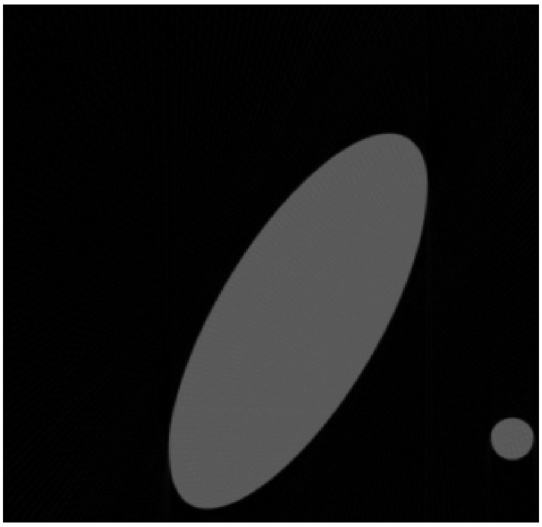

Figure 3. Reconstruction of Annex 2

3.3 Determine position and geometry

First, we can see from the above that the initial illumination angle is 29 degrees from the long axis direction of the ellipse. We want to start the calculation from the right vertical injection. Therefore, the first 60 columns of data in Annex 3 are mirrored and added to the data of 180st. Then each group of data is moved forward one time, finally, the effect of the square is achieved. The figure below is the figure after the figure 5 is squared. The same problem can be solved by solving the problem three.

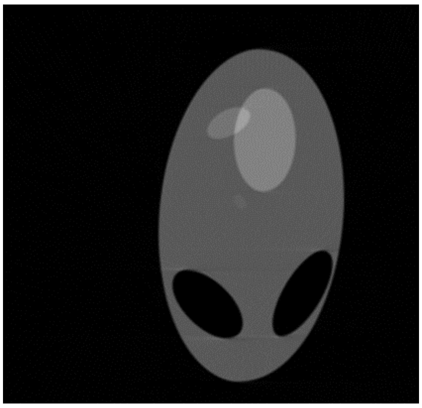

Figure 4. Geometric image of the unknown medium

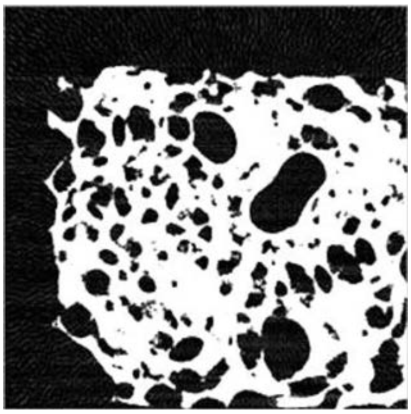

Figure 5. Geometric image of problem three unknown media

3.4 Determination of absorption rate

It is known from Annex 1 that the absorption rate of all points is 1 in the range of the template, and the absorption rate of all points is 0 outside the range of the template. The absorption rate is reflected in the reconstructed figure is its color. So we extract the gray mean of the known model and the reconstructed figure formed in Annex III and then use the proportional relationship of the gray value to calculate the absorption rate of each point. Finally, the obtained absorption rate matrix is used to draw a three-dimensional image, which is highly consistent with the reconstructed image.

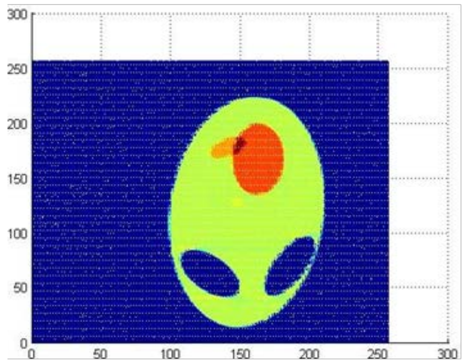

Figure 6. Problem 2: 2D plot of the absorptivity of an unknown medium

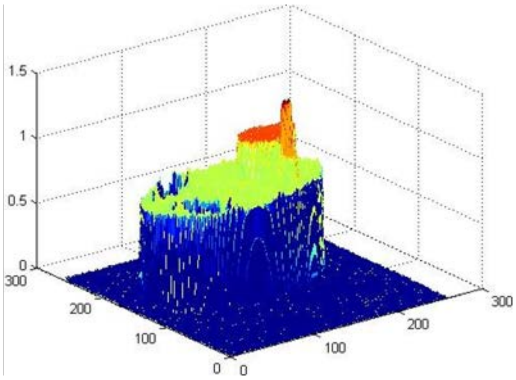

Figure 7. Problem 2: 2D plot of the absorptivity of an unknown medium

The coordinates given in Annex 4 are based on the lower left corner of the square pallet as the coordinate origin, and the coordinate unit is mm. Therefore, after converting to the coordinates corresponding to the absorption rate, the calculated values of the 10 point absorption rates are as follows:

Table 1. Absorption rate of ten given points

x/mm	10.0000	34.5000	43.5000	45.0000	48.5000	50.0000	56.0000	65.5000	79.5000	98.5000
y/mm	18.0000	25.0000	33.0000	75.5000	55.5000	75.5000	76.5000	37.0000	18.0000	43.5000
Absorption rate value	0	0.0080	0.7280	0	0.7200	0	0	1.0480	0.0160	0.0080

For the third problem, an absorption rate matrix is generated. After testing, the absorption rate matrix is compared with the reconstruction matrix, and the degree of agreement is very high, which also proves the correctness of our method. At the same time, after the above treatment, the absorption rate of 10 points can be obtained:

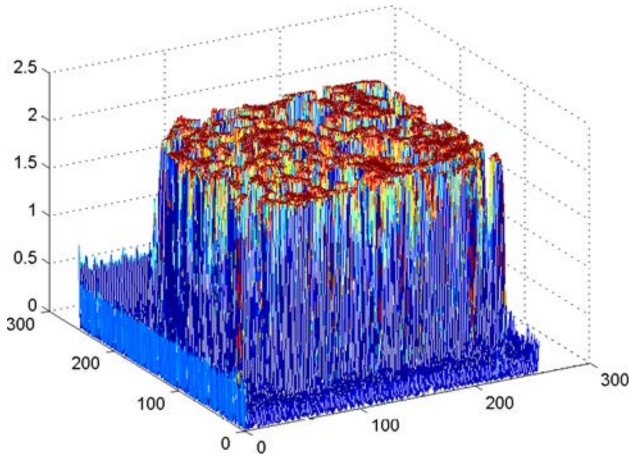

Figure 8. Absorption rate reverse verification chart

Table 2. Absorption rate of ten given points

x/mm	10.0000	34.5000	43.5000	45.0000	48.5000	50.0000	56.0000	65.5000	79.5000	98.5000
y/mm	18.0000	25.0000	33.0000	75.5000	55.5000	75.5000	76.5000	37.0000	18.0000	43.5000
Absorption rate value	0.0320	1.7520	0.1040	0.0640	2.0400	1.4320	0.7440	0	1.4800	0.0880

4. Summary

For the calibration of CT system parameters, this paper determines the rotation center, cell spacing and 180 rotation angles of the system template according to the attenuation law of X-ray and combined with known data. The filtered back projection method based on Radon transform realizes image reconstruction, which has the advantages of high precision and small calculation amount, and can obtain better reconstructed images. However, there are star-shaped artifacts at the edge of the reconstructed image, which cannot completely eliminate its influence. This is also the direction that we need to conduct further research. The transmitting-receiving device adopts a parallel beam scanning mode, and the rays generated by the transmitter are all discrete and thus cause the projection information to be discontinuous to generate a certain error, resulting in inaccurate positioning, low precision and stability, and further we will further follow Optimizing it to the equiangular sector scan mode [11] shortens the scan time and makes the image reconstruction closer to the real situation.

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